Doubling Down on Debt: Limited Liability as a Financial Friction

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This Paper

- Goals of this paper:
  - Study distortions arising from limited liability and existing debt on real **investment** relative to "efficient" levels
  - Investigate how these distortions are affected by **equity payouts**
- Simple model of firm investment
  - A single firm protected by limited liability and facing default risk
  - Intensive investment + intensive equity payouts + capital structure
  - Firm faces (one-shot/repeated) investment opportunities
- Only source of financial friction is **limited liability**
  - **No moral hazard**: complete information, no theft, perfect monitoring
  - **No gambling-for-redemption**, risk shifting, etc.
  - Clarity on ownership of all cashflows. Competent accountants!
- Empirical evidence on equity payouts & investment by leverage

Characterizing Limited Liability

Limited liability is a protection of equity holders' non-firm assets (including human capital) from creditors

But that doesn't necessarily mean there is a friction. However,

- In all of its forms, the central **financial friction** is that limited liability leads to a **commitment** problem due to ex-post incentives
- Equity holders raising debt cannot credibly promise to either:
  1. never default
  2. pay debt holders personal assets outside of the firm—thereby making punishments more effective

Literature on Financial Frictions

Models of financial frictions usually **limited liability** (sometimes implicit and hidden) + incomplete markets

- + **private information** (e.g., Bernanke et al. (1999) or Clementi and Hopenhayn (2006))
- + **inalienable human capital** (e.g., Kiyotaki and Moore (1997), and Albuquerque and Hopenhayn (2004))
- + **risk-taking incentives** (e.g., Jensen and Meckling (1976) and Vereshchagina and Hopenhayn (2009))
- + **limited enforcement** (e.g., Buera et al. (2011), Moll (2014))
- + **nothing!** (this paper, also idiosyncratic prices)

Broader Literature

- Primarily in the spirit of micro-founding frictions in macro-finance **heterogenous** across productivity, debt, or leverage
  - Papers on previous slide + many other classics
- Recent macro literature on macro-distortions from debt
  - e.g. Lian and Ma (2021), Atkeson et al. (2017), Crouzet and Tourre (2020), Acharya and Plantin (2019), Kalemli-Ozcan et al. (2018), Jungherr and Schott (2021)
- Strong connections with (and differences from) sovereign default
  - e.g. Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2013), Hatchondo et al. (2016), and, especially, Aguiar et al. (2019)
- Complementary to literature on corporate finance, debt overhang, and "leverage racheting"
  - e.g. Myers (1977), Leland (1998), Moyen (2007), and Diamond and He (2014), Admati et al. (2018), DeMarzo and He (2020)

Summary of Results

- Highly leveraged firms have incentives to further increase leverage
  - One-shot investment: **overinvestment** if any preexisting liabilities
  - Repeated investment: **overinvestment** by **high leverage** firms
  - Mechanism generates **heterogeneity of distortions** on real investment
- Financial friction: **double-selling cashflows in default**
  - Distinct from risk-shifting
  - Dilution of pre-existing liabilities (but not collateral claims)
  - Time-consistency: incentives to “double-sell” increase price of debt
- **Equity payouts** are efficient way to **dilute existing debt-holders**
  - One-shot: Mitigate inefficient overinvestment
  - Repeated: Under-investment for low-liability firms ($\uparrow$ prices)
- Distortion from **time-inconsistency and market incompleteness**, not due to information economics or moral hazard

Outline

1. Minimal model of one-shot investment opportunity
   - Given liabilities at time of investment
   - Agnostic on source and price of old liabilities
2. Analysis and characterization of new mechanism
   - **How** equity holders can benefit without any "information economics"
3. Model with repeated investment opportunities
   - Now sequence of liabilitiess
   - Lets us look at the **dynamic distortion** from the incentive
4. Empirical evidence and quantitative analysis (see paper)

MODEL WITH DEFAULTABLE DEBT

Model of a Firm Investment Decision

Starting analysis at $t = 0$ a (pre-existing) firm has:

- State (Z, L) at point of one-shot investment opportunity
  - **Snapshot in time:** Source of pre-existing L doesn't matter
- Assets-in-place/productivity/capital, Z
  - Profits before debt service also Z, discounted at rate r
- Pre-existing liabilities with PV of promised payouts $L \geq 0$
  - The old price of L could have taken into account this opportunity
  - In repeated, we will examine where it may have come from and time-inconsistency issues induced by this mechanism
  - Exogenous reasons? For example, initial L might come due to collateral constraints

Investment and Evolution of Z

- Assume operating profits, Z, follow Geometric Brownian Motion:

$$\mathrm{d}Z(t) = \sigma Z(t)\mathrm{d}\mathbb{W}(t)$$

- Enterprise value is expected present value of cash flows, $\frac{Z}{r}$
  - Careful with accounting of **claims** of all **cash flows**
- Invest in g such that $Z \to (1+g)Z$
  - Assume convex cost: $q(g)Z = \frac{\zeta}{2}g^2 Z$
- Let the **optimal investment choice** of the firm be $g(Z, L)$
- Preview of repeated-investment model
  - Arrival rate of opportunities makes (Z, L) a controlled jump-diffusion

Financing the Investment

- Full-information, competitive price-taking agents, no market-power
- Assume firm can sell **defaultable consol bonds** with an embedded claim to the liquidation value of the firm for each bond
  - i.e. **secured** bond: L has claims in default at a fixed proportion
  - For the asset and basic pricing approach see Leland (1998)
- Firm may use a mix of equity and debt financing
  - Proportion of $q(g)Z$ **financed by debt** is a chosen ψ
- Firm can make direct **equity payouts** to themselves, $M \in [0, \kappa Z]$
  - Constraint $\kappa \geq 0$ captures institutional and legal constraints
  - Baseline is $\kappa = 0$, i.e. all financing must go into firm assets.
- Defaultable consol: pays 1 until default then a claim in liquidation

$$\mathbf{P(Z,L)} = \underbrace{\mathbf{P^C(Z,L)}}_{\textbf{Coupons}} + \underbrace{\mathbf{P^B(Z,L)}}_{\textbf{Bankruptcy Claim}}$$

Summary of Parameters and Decisions

- Only **two** essential parameters for mechanism (+ one scale)
  - r: **risk-free interest rate**
  - σ: **volatility of operating profits**
  - $q(\cdot)$: convex cost, assume quadratic $q(g) \equiv \frac{\zeta g^2}{2}$
    - ζ is a largely an uninteresting scale parameter
  - κ: constraint on equity payoffs $= 0$ baseline
- Decisions of equity holders is to choose
  - Investment size, g, debt financing proportion ψ, equity payouts M
  - Continuous **default** policy comparing PV of liabilities to PV of profits

$$\max\{\mathbf{0}, \mathbf{V}(\mathbf{Z}, \mathbf{L})\}$$

- Decisions of competitive new debt holders
  - **Pricing** of new debt when financing. Competitive, full information
  - Given equity holders investment, default decisions, equity payouts
- Passive old debt holders:
  - Recall, no stand taken on prices for original L

Investment Choice Summary

Equity holders take the equilibrium bond price $P(\cdot)$ as given and solve

$$V^*(Z,L) = \max_{\substack{g\geq 0 \\ \psi\in[0,1] \\ 0\leq M\leq \kappa Z}} \Big\{ \overbrace{V(\underbrace{(1+g)Z}_{=\hat{Z}},\hat{L})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)Z}^{\text{Equity Financed}} + \overbrace{M}^{\text{Payouts}} \Big\}$$

$$\text{s.t. } \underbrace{P(\hat{L},Z,L,g,\phi,M)}_{\text{Equilibrium Price}} \underbrace{\left(\hat{L}/r - L/r\right)}_{\text{New Bonds}} = \underbrace{\psi q(g) Z}_{\text{Debt Financed}}$$

- The post investment liabilities, $\hat{L}(\cdot)$ come from pricing of new debt
- Induces a $(Z,L) \to (\hat{Z},\hat{L})$ jump

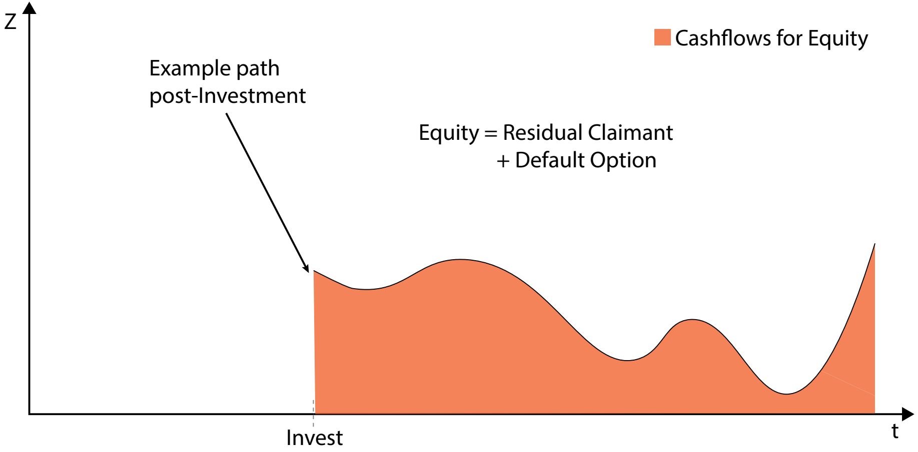
Definition (First-Best Investment)

We define the first-best undistorted investment, g^u, as investment that maximizes the net present value of the firm. That is,

$$g^u(Z) \equiv \arg\max_g \left\{ \overbrace{V((1+g)Z, 0)}^{\text{Post-Investment Equity}} - \overbrace{q(g)Z}^{\text{Equity Financed}} \right\} \tag{1}$$

i.e. equity holders have no debt and deep pockets

Example Cashflow, All Equity



- All cashflows are fairly priced
- Consider example path, valuations are expected PDV
- Would Modigliani-Miller hold? (i.e. capital structure non distorting)

Post-Investment Problem

- Firm with (Z, L) has an optimal stopping problem,

$$rV(Z,L) = Z - rL + \frac{\sigma^2}{2} Z^2 \partial_{ZZ} V(Z,L)$$
$$V(\underline{Z}(L), L) = 0$$
$$\partial_Z V(\underline{Z}(L), L) = 0$$

- The solution is a **default decision rule** $\underline{Z}(L)$
- Equity holders optimally walk away when they reach negative equity
  - i.e., $V(Z,L) \leq 0$ when $Z \leq \bar{Z}(L)$

Default Decision and Equity Value

Proposition (Continuation Value and Default Choice)

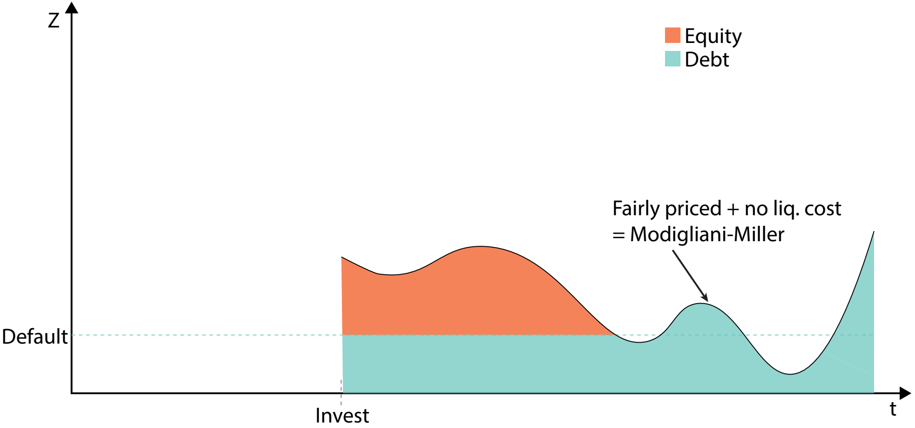
The normalized equity value with $\ell \equiv L/Z$ *is,*

$$\frac{V(Z,L)}{Z} = \frac{1}{r} - \ell + \overbrace{\ell \underbrace{\frac{\chi}{\eta+1}\ell^{\eta}}_{\equiv s(\ell)}}^{\textit{Option Value}}$$

η *and* χ *functions of* r *and* σ*. And*

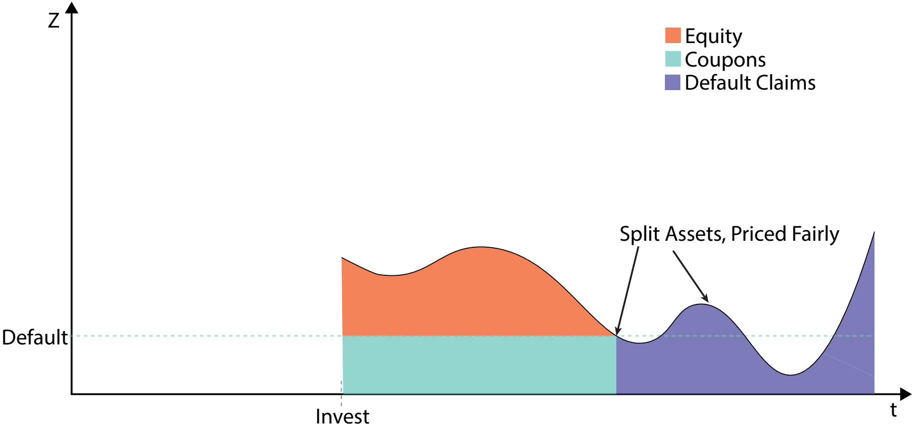
$$\frac{Z}{\underline{Z}(L)} = \frac{\eta+1}{\eta}\frac{1}{r\ell}$$

Would Modigliani-Miller Hold?



- This is a single (fairly priced) Z path, agents use EPDV
- Modigliani-Miller manifests as indeterminacy of the default threshold (considering all paths of Z in the EPDV)

Decoupling Liabilities from Default Claims



- Default claims could be sold by firm directly, or stripped by claimant
- Even if in the same asset, valuable to separate for intuition

Prices and Spreads

Proposition (Price of a Defaultable Consol)

For a firm with state $\ell = L/Z$ with only defaultable consol bonds,

$$p(\ell) = 1 - \underbrace{s(\ell)}_{\textit{Spread}} = \underbrace{(1-(1+\eta))s(\ell)}_{\equiv p^C(\ell)} + \underbrace{\eta s(\ell)}_{\equiv p^B(\ell)}$$

- $\uparrow \ell$ then $p^C(\ell) \downarrow$ and $p^B(\ell) \uparrow$
- But overall, $\uparrow \ell$, then $p(\ell) \downarrow$ and $s(\ell) \uparrow$.
- No coincidence: recall **option value of default** in $v(\ell)$ solution

$$\frac{V(Z,L)}{Z} \equiv v(\ell) = \frac{1}{r} - \ell + \overbrace{\ell \underbrace{\frac{\chi}{\eta+1}\ell^{\eta}}_{\equiv s(\ell)}}^{\textit{Option Value}}$$

- But how can firm manipulate this term and benefit?

Firm Investment

The problem of a firm with $\ell \equiv L/Z$ is to choose $(g, \psi, \hat{\ell}, m)$ such that,

$$v^*(\ell) = \max_{\substack{g \geq 0 \\ \psi \in [0,1] \\ 0 \leq m \leq \kappa}} \left\{ \overbrace{(1+g)v(\hat{\ell})}^{\text{Post-Investment Equity}} - \overbrace{(1-\psi)q(g)}^{\text{Equity Financed}} + \overbrace{m}^{\text{Payouts}} \right\}$$

$$\text{s.t.} \quad \underbrace{p(\hat{\ell})}_{\text{Bond Price}} \underbrace{((1+g)\hat{\ell} - \ell)}_{\text{New Bonds}} = \underbrace{\psi q(g)}_{\text{Debt Financed}} + \underbrace{m}_{\text{Payouts}}$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell})$$

The first-best investment solves

$$g^u \equiv \arg\max_g \left\{ \overbrace{(1+g)v(0)}^{\text{Post-Investment Equity}} - \overbrace{q(g)}^{\text{Equity Financed}} \right\}$$

ANALYSIS

Rewrite Equity Holder's Problem

$$v^*(\ell) = \max_{\substack{g,\hat{\ell}\geq 0 \\ \psi\in[0,1] \\ 0\leq m\leq\kappa}} \left\{ \overbrace{\frac{1+g}{r} - q(g)}^{\text{Undistorted}} - p(\hat{\ell})\ell \right\}$$

$$\text{s.t. } p(\hat{\ell})((1+g)\hat{\ell} - \ell) = \psi q(g) + m$$

$$p(\hat{\ell}) \geq p^B(\hat{\ell})$$

- The first-best investment, g^u, is the unique solution to $\frac{1}{r} - q'(g^u) = 0$
- Modigliani-Miller Theorem holds if $\ell = 0$
- If $\ell > 0$: $\hat{\ell} \downarrow$ decreases v^* since $p(\hat{\ell}) \downarrow$ in $\hat{\ell}$
- Symmetrically: incentive to increase $\hat{\ell}$ independent of investment
- Payoffs, m, not directly in objective. Must manipulate $\hat{\ell}$

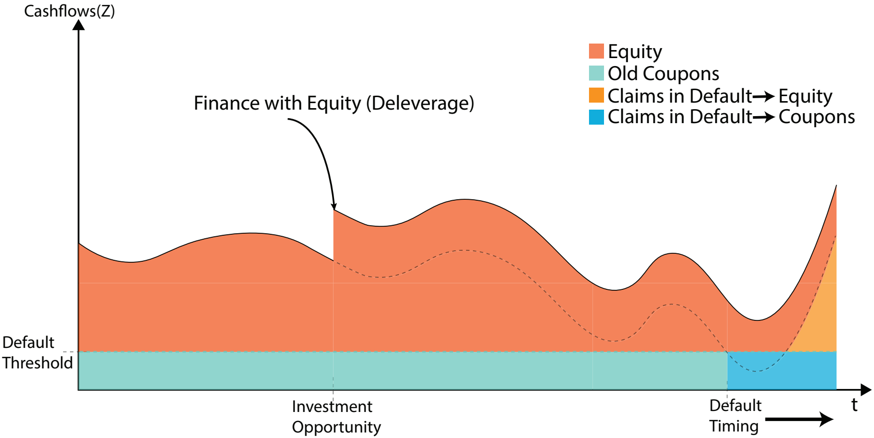
Characterizing Over/Under Investment

Proposition

Suppose that $\kappa = 0$ and $\ell > 0$. If equity holders can

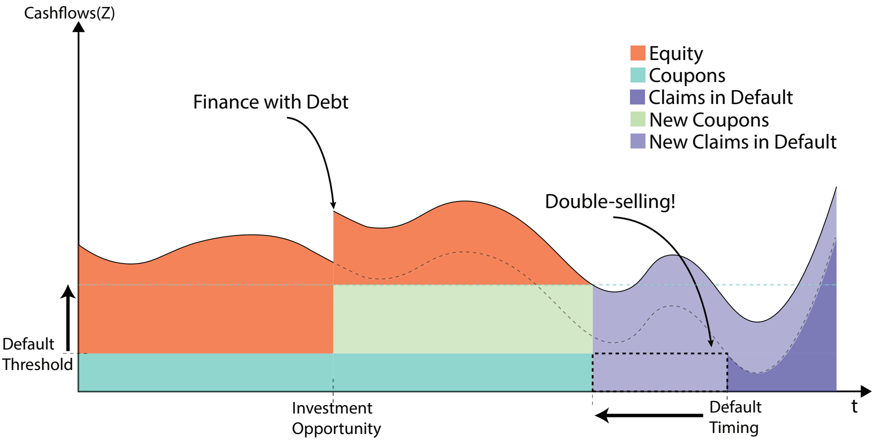
1. *only use* **equity financing** *then they* **underinvest**
2. **choose financing** *optimally then then finance with debt and* **they overinvest**

Equity Financing Decreases the Option Value of Default



- Deveraging: Same default threshold, pays coupons longer
- Converts old claims in default to coupons, but can't benefit

Debt Financing Dilutes Existing Claims to Coupons



- Due to increased leverage, dilutes existing debt holders and **double-selling** some of their promised coupon payments
- Converts old coupon claims to new default claims!
- Increased leveraged is a commitment to earlier default

Equity Payouts "Efficiently" Increase Leverage

Proposition

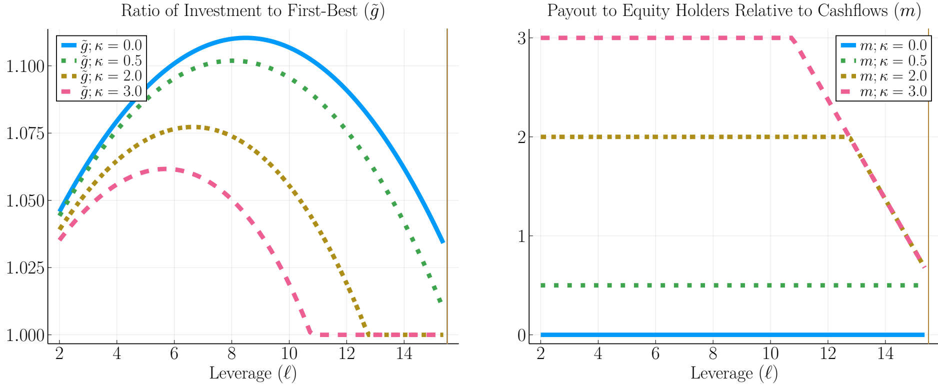
For g^*, m^*, ψ^* optimal choices, there exists $\underline{\kappa}$ such that

1. *If $\kappa < \underline{\kappa}$ then equity holders*
   1. **overinvest**, *that is $g^* > g^u$*
   2. *make* **payouts to the constraint**, *that is $m^* = \kappa$*
2. *If $\kappa \geq \underline{\kappa}$ then equity holders*
   1. **invest the first-best** *amount, that is $g^* = g^u$*
   2. *make* **payouts to up to the default threshold** $m^* < \kappa$

The threshold $\underline{\kappa}$ is $\downarrow$ in ℓ and r, and $\uparrow$ in σ.

- **Separately** dilute existing coupons & maximize enterprise value
- Sell **new collateral claims** to maximized firm value— profiting on old-coupon cashflows through m and $g > g^*$ (if constrained by κ)
- Reminder: still looking at the ex-post incentives

Investment Relative to First-Best for $\kappa \geq 0$



- Investment relative to first-best $\tilde{g} \equiv g/g^u$
- $\kappa = 0$ captures strict and $\kappa = 3.0$ lax constraints

MODEL WITH REPEATED INVESTMENT

Arrival of Investment Opportunities

- Time-inconsistency suggests repeated version may be interesting
- Prices will reflect lack of ability to commit, and will distort asymmetrically
- Arrival rate $\lambda \geq 0$ of investments where $\lambda = 0$ nests one-shot
- Same problem of optimal investment time, given dynamic ℓ
- $v(\cdot)$ and $p(\cdot)$ consider future investments

Evolution of Liabilities and Cash-Flows

- $\mathbb{N}(t)$ is a Poisson process with intensity $\lambda \geq 0$.
  - $g(Z(t^-), L(t^-))$ is the optimal investment choice
  - $\hat{L}(Z(t^-), L(t^-))$ is the corresponding post-investment liabilities
- Cash-flows, Z, now follows a jump-diffusion

$$\mathrm{d}Z(t) = \sigma Z(t)\mathrm{d}\mathbb{W}(t) + g(t^-)d\mathbb{N}(t)$$

- Liabilities, L, follows a pure jump-process

$$\mathrm{d}L(t) = (\hat{L}(t^-) - L(t^-))\mathrm{d}\mathbb{N}(t),$$

- We can solve the equilibrium numerically

Proposition (Repeated Investment)

Solution: normalized equity value $v(\ell)$, price $p(\ell)$, policies $\{g(\ell), m(\ell), \psi(\ell), \hat{\ell}(\ell), \bar{\ell}\}$ such that

1. *Given $v(\ell)$ and $p(\ell)$, the policies solve the firm's investment problem*
2. *Given $p(\ell)$ and the policies, $v(\ell)$ solves the DVI*

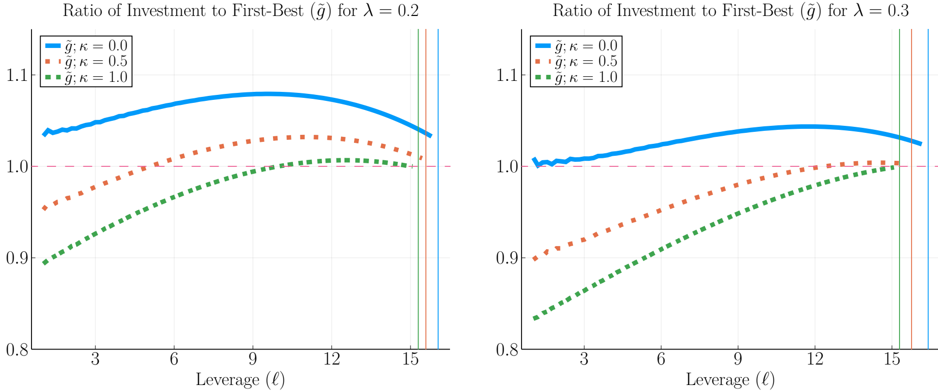
$$0 = \min\{rv(\ell) - \frac{\sigma^2}{2}\ell^2 v''(\ell) - \lambda\left(v(\hat{\ell}(\ell)) - v(\ell)\right) - (1 - r\ell), v(\ell)\}$$

3. *Default threshold $\bar{\ell}$ is optimal, indifference point of the DVI*
4. *Given $v(\ell)$ and the policies, $p(\ell)$ solves BVP (i.e. doesn't control $\bar{\ell}$)*

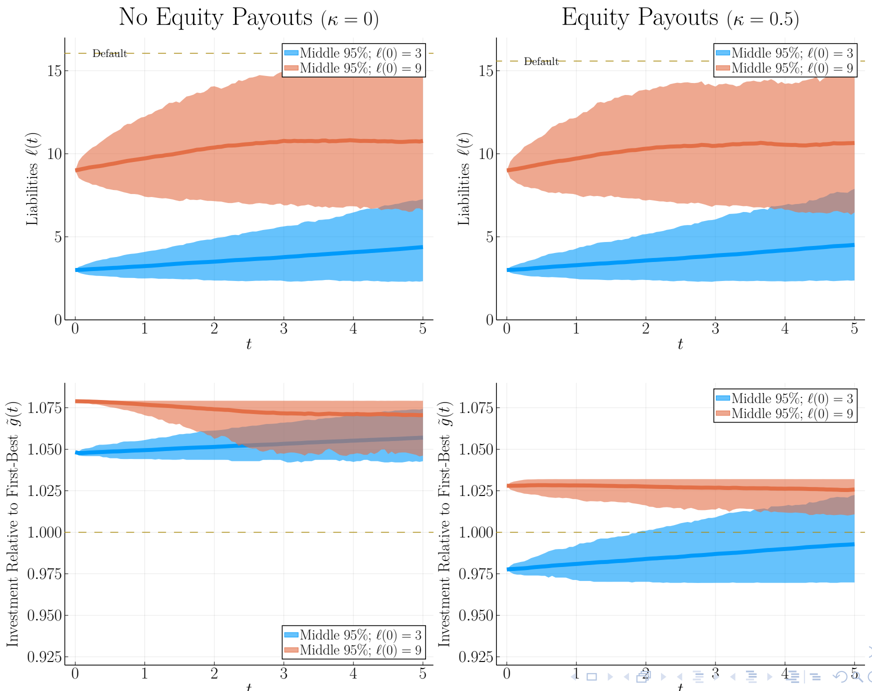
$$rp(\ell) = r + \sigma^2 \ell p'(\ell) + \frac{\sigma^2}{2}\ell^2 p''(\ell) + \lambda\left(p(\hat{\ell}(\ell)) - p(\ell)\right)$$

$$p(\bar{\ell}) = \frac{v(0)}{\bar{\ell}}$$

Investment Relative to First-Best for $\lambda \geq 0$



- $\kappa > 0$ still mitigates over-investment, but can cause under-investment
- $\kappa = 0$ no equity payouts, $\kappa = 1.0$ laxer constraint


No Equity Payouts ($\kappa = 0$)
Equity Payouts ($\kappa = 0.5$)
Default
Middle 95%; $\ell(0) = 3$
Middle 95%; $\ell(0) = 9$
Liabilities $\ell(t)$
Investment Relative to First-Best $\tilde{g}(t)$
t

Conclusion

- Strong incentives to increase leverage with preexisting debt
  - Leads to over-investment in a one-time investment model
  - When equity payouts are allowed, "efficient" leveraging mitigates over-leveraging.
- New financial friction induced by limited liability: **double-selling claims in default**
- The force remains in a repeated model
  - Repeated investment make debt more expensive *because* of this friction
  - The ease of dilution from equity payoffs makes them especially distortionary for low leverage firms
- Extensions in paper: seniority, bankruptcy costs, unsecured debt
- Policy Discussion in Paper: empirical evidence on equity payoffs/overinvestment consistent with the model

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